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k-Coterie and Coterie Join Operation

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Abstract. In distributed systems, quorum-based algorithms are used to access shared resources in a mutually exclusive way because these algorithms are more resilient to process and communication failures. Based on resource allocation, the quorum system for the mutual exclusion problem is a coterie and the quorum system for the *k*-mutual exclusion problem is a *k*-coterie. There are two types of coteries and *k*-coteries which are dominated and nondominated. The availability of a coterie and a *k*-coterie need to be considered in an error-prone system. Therefore, *k*-coterie and coterie join operation is presented to build a new larger *k*-coterie so that the availability of a *k*-coterie increases. Through an analysis of availability is obtained that a *k*-coterie C is nondominated if and only if for any probability *p*, the sum of the (*k*,1)-availability and the (*k, k*)- dual availability of a coterie C equals one.

1. Introduction

A distributed system is a set of n processes that communicate by exchanging messages. This system has shared resources that must be accessed in a mutually exclusive way. The process is said to be in the critical section when the process is accessing resources. How to control access from a shared resource so that at most one process in the critical section is an important problem known as the problem of mutual exclusion (mutex). Quorum-based algorithms are used to achieve mutually exclusive access to critical sections because this algorithm can tolerate process and communication failures. The basic idea of a quorum-based algorithm is to collect permissions to form a quorum to enter the critical section. Mutex is guaranteed if there is only one quorum that can be formed at any instance [1].

Quorum systems serve as a basic tool that provides a uniform and reliable way to achieve coordination between processors in a distributed system. Quorum systems are defined by the following chronology. A set system is a collection of sets $\mathbb{C} = \{Q_1, \dots, Q_m\}$ under $U = \{u_1, \dots, u_n\}$. A set system is said to satisfy the intersection properties if every two sets of $Q_i, Q_j \in \mathbb{C}$ has a non-empty intersection. The set system with intersection properties is known as the quorum system, and the set in the system is called the quorum. The quorum system is said to satisfy minimality property if every two sets is not a subset of one another. The quorum system that has a minimum of properties is called coterie [2]. In other words, coterie is a set system that has intersections and minimalities. Apart from intersection and minimality properties, the coterie is extended with disjoint properties. A set system is said to satisfy the mutually exclusion if every two sets in the system have empty intersections. So, a coterie that has at most k disjoint set is called k-coterie. In resource allocation problems, a coterie is used for mutex problem and k-coterie is used in the k-mutex problem.

In 1985 Garcia-Molina and Barbara classified coteries into two types: dominated and nondominated coteries. Nondominated coteries are more resilient to process and communication failures than dominated coteries. Because the availability of distributed systems is better if nondominated coteries are used. A function called a coterie transformation is defined to derive a new nondominated coterie

from existing ones [3]. In 1992, Nielsen and Mizuno discussed coterie transformation of integrating coterie where a new coterie can be constructed by replacing an element in quorum of the first coterie by elements in a quorum in the second coterie [4]. In 2016, Ishak researched about how to construct nondominated coteries using the join operation [5]. In 2018, the development of the research was how the join operation may be applied in k-coteries by Muhlis [6].
This study discusses about k-coterie and coterie join operation to construct a new larger k-coterie so the availability of k-coterie increases. The availability of k-coterie needs to be considered in an error-prone system because the highest availability of k-coterie is more resilient to process and communication failures in a system.

2. Distributed Resource Allocation Problem

Distributed resource allocation problem included in this study are mutex and k-mutex problems. A mutex is a condition of a distributed system where there is only one process can access the resource at any given time. While a k-mutex is a condition in the distributed system in which there are at most k processes that can access the resource at any given time. So as to ensure the condition of mutex and k-mutex problems used an algorithm based on a quorum.

In the mutex problem is used coterie scheme to ensure only one process can access the resource at any given time. In which a process must obtain permission from all processes in a quorum of the coterie to enter the critical section and each process does not give permission more than one process at any given time. Since each two intersections of quorum has not empty, then more than one process cannot enter the critical section at any given time.

In the k-mutex problem is used k-coterie scheme to ensure at most k processes that can access the resource at any given time. Where k process must obtain permission from all processes in the quorum of the k-coterie to enter the critical section simultaneously and each process does not give permission over k process at any given time. Because there is a disjoint k-quorum, then at most k process can enter the critical section at a certain time [7].

3. Quorum Systems

Suppose that U is a universe set of processes in a system.
Definitions 1. A set system $\mathbb{C} = \{Q_1, \dots, Q_m\}$ is a collection of subsets $Q_i \subseteq U$ of a finite universe U. A quorum system is a set system $\mathbb{C}$ that has the intersection property [2].

3.1 Coteries

A coterie is a quorum system $\mathbb{C}$ that has the minimality property. In addition, a coterie can be defined as follows:
Definition 2. A nonempty collection of sets $\mathbb{C}$ is a coterie under U if and only if the following properties are satisfied:
1) intersection property: $\forall Q_1, Q_2 \in \mathbb{C},\ Q_1 \cap Q_2 \neq \emptyset$,
2) minimality property: $\forall Q_1, Q_2 \in \mathbb{C},\ Q_1 \not\subset Q_2$.
The sets $Q_1, Q_2 \in \mathbb{C}$ are called quorums [4].
Example 1: Let $\mathbb{C} = \{\{a,b\},\{a,c\},\{b,c\}\}$ be a coterie of the universe $U = \{a,b,c\}$ because each pair of quorums in $\mathbb{C}$ have nonempty intersections and there is no quorum which is a subset or equal to other quorum.

Definitions 3. Let $\mathbb{C}_1$ and $\mathbb{C}_2$ be coteries in the same universe U. $\mathbb{C}_1$ dominates $\mathbb{C}_2$ if and only if
1) $\mathbb{C}_1 \neq \mathbb{C}_2$,
2) $\forall Q_2 \in \mathbb{C}_2,\ \exists Q_1 \in \mathbb{C}_1\ s.t.\ Q_1 \subseteq Q_2$.
A coterie $\mathbb{C}$ in a universe U is dominated if there is another coterie in the universe U that dominates $\mathbb{C}$. If there is no such coterie, then $\mathbb{C}$ is a nondominated coterie [4].

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Example 2: Let coteries $\mathbb{C}_1 = \{\{a,b\},\{a,c\},\{b,c\}\}$ and $\mathbb{C}_2 = \{\{a,b\},\{b,c\}\}$ be coteries in the universe $U = \{a,b,c\}$. A coterie $\mathbb{C}_1$ dominate $\mathbb{C}_2$ because $\mathbb{C}_1 \neq \mathbb{C}_2$ and for any quorum Q_2 in $\mathbb{C}_2$ there is a quorum Q_1 in $\mathbb{C}_1$ such that Q_1 equals Q_2.

A nondominated coterie, like $\mathbb{C}_1$, is always better than a dominated coterie, like $\mathbb{C}_2$, for if a quorum can be formed in the dominated coterie then a quorum can be formed in coterie that dominates. A coterie is nondominated if no other coterie can dominate it. A nondominated coterie is a candidate to achieve the highest availability, which is the chance that a quorum can be formed in an error-prone environment. So, nondominated coteries are a center of concentration if the tolerance of failure is one of the main concerns.

Theorem 1 makes it easier to determine if a coterie is dominated [8].

Theorem 1. Let $\mathbb{C}$ be a coterie under U. Then, $\mathbb{C}$ is dominated if only if there is a set $X \subseteq U$ such that
1) $\forall Q \in \mathbb{C}\,,\ Q \nsubseteq X$,
2) $\forall Q \in \mathbb{C}\,,\ Q \cap X \neq \emptyset$.

Example 3: Let $\mathbb{C} = \{\{a,b\},\{b,c\}\}$ be a coterie under $U = \{a,b,c\}$. So, $\mathbb{C}$ is dominated because $\exists\, \{b\} \subseteq U$ s.t.
- $\{a,b\} \nsubseteq \{b\}$ and $\{a,b\} \cap \{b\} = \{b\}$
- $\{b,c\} \nsubseteq \{b\}$ and $\{b,c\} \cap \{b\} = \{b\}$

Or because $\exists\, \{a,c\} \subseteq U$ s.t.
- $\{a,b\} \nsubseteq \{a,c\}$ and $\{a,b\} \cap \{a,c\} = \{a\}$
- $\{b,c\} \nsubseteq \{a,c\}$ and $\{b,c\} \cap \{a,c\} = \{c\}$

3.2 k-coteries

Definition 4. A non-empty set system $\mathbb{C}$ of non-empty subsets Q of U is called a k-coterie if and only if all of the following three conditions hold:
1) Non-intersection property:
$\forall h\ (1 \leq h < k)$ quorums, $Q_1, \ldots, Q_h \in \mathbb{C}, Q_i \cap Q_j = \emptyset\ \forall i,j,\ 1 \leq i \neq j \leq h$,
$\exists Q \in \mathbb{C}\ s.t.\ Q \cap Q_i = \emptyset\ \forall i, 1 \leq i \leq h$.
2) Intersection property:
$\forall l > k$ quorums, $Q_1, \ldots, Q_l \in \mathbb{C}, \exists Q_i, Q_j\ s.t.\ Q_i \cap Q_j \neq \emptyset\ \forall i,j, 1 \leq i \neq j \leq l$.
3) Minimality: $\forall Q_i, Q_j \in \mathbb{C},\ \ Q_i \nsubseteq Q_j$.

An element Q of k-coteries $\mathbb{C}$ is called a quorum. Note that 1-coterie is a coterie, and because of that, a k-coterie concept is an extension of a coterie [7].

By non-intersection property, if there is an empty entry of a critical section, then a waiting process to enter a critical section can be processed. The intersection property ensures that no more than k processes can form a quorum simultaneously, so that no more than k processes can access the critical section at the same time. The minimality property for k-coterie is used to increase efficiency [9].

Example 4: Let $\mathbb{C} = \{\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$ be a 2-coterie in the universe $U = \{a,b,c,d\}$ because the set system satisfies the following three conditions:
1. Non-intersection property: Take any $Q = \{a,b\} \in \mathbb{C}$, there are elements $Q' = \{c,d\} \in \mathbb{C}$ such that $Q \cap Q' = \{a,b\} \cap \{c,d\} = \emptyset$.
2. Intersection property: Take any three elements $\{a,b\},\{c,d\},\{b,d\} \in \mathbb{C}$, there is a pair of elements $\{c,d\},\{b,d\}$ such that $\{c,d\} \cap \{b,d\} = \{d\}$
3. Minimality: Take any $\{a,c\},\{b,c\} \in \mathbb{C}$ such that $\{a,c\} \nsubseteq \{b,c\}$.

Definitions 5. Let $\mathbb{C}_1$ and $\mathbb{C}_2$ be k-coteries in the same universe U. $\mathbb{C}_1$ dominates $\mathbb{C}_2$ if only if
1) $\mathbb{C}_1 \neq \mathbb{C}_2$,
2) $\forall Q_2 \in \mathbb{C}_2, \exists Q_1 \in \mathbb{C}_1\ s.t.\ Q_1 \subseteq Q_2$,

A k-coterie $\mathbb{C}$ in a universe U is dominated if there is another k-coterie in the universe U that dominates $\mathbb{C}$. If there is no such k-coterie, then $\mathbb{C}$ is a nondominated k-coterie [10].

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Example 5: Let

$$\mathbb{C}_1 = \{\{a\},\{b,c\},\{b,d\},\{c,d\}\}$$

and

$$\mathbb{C}_2 = \{\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$$

be 2-coterie in the universe $U = \{a,b,c,d\}$. So $\mathbb{C}_1$ dominates $\mathbb{C}_2$ because $\mathbb{C}_1 \neq \mathbb{C}_2$ and for any quorum Q_2 in $\mathbb{C}_2$ that there is a quorum Q_1 in $\mathbb{C}_1$ such that Q_1 subsets or equals to Q_2. Therefore, there are no another 2-coterie in the universe U that dominates $\mathbb{C}_1$, so $\mathbb{C}_1$ is a nondominated 2-coterie in the universe U.

The simple method to determine a dominated k-coterie is in the theorem 2 [10].

Theorem 2. A k-coterie $\mathbb{C}$ in a universe U is dominated if and only if there is a set $X \subseteq U$ such that

1) $\forall Q \in \mathbb{C}, Q \nsubseteq X$,
2) $\forall k, \{Q_1, \dots, Q_k\} \subseteq \mathbb{C}, Q_i \cap Q_j = \emptyset \ \forall i,j, 1 \leq i \neq j \leq k, \ Q \cap Q_i = \emptyset \ \exists i$.

Example 6: Let $\mathbb{C} = \{\{a\},\{b,c\},\{b,d\}\}$ be a 2-coterie in a universe $U = \{a,b,c,d\}$ is dominated because there are set $\{c,d\} \subseteq U$ such that

1. Take any $\{b,d\} \in \mathbb{C}, \{b,d\} \nsubseteq \{c,d\}$.
2. For any collection of two disjoint quorums, $\{\{a\},\{b,d\} \subseteq \mathbb{C}\}$, then $\{c,d\} \cap \{b,d\} = \{d\}$

A nondominated k-coterie is more resilient to interference. The reason can be understood through the example 7.

Example 7: Let $\mathbb{C}_2$ be a k-coterie that dominated by $\mathbb{C}_1$. Thus, each quorum in $\mathbb{C}_2$ contain a quorum of $\mathbb{C}_1$. Let $U = \{1,2,3,4,5,6\}$ can be construct the following 2-coterie $\mathbb{C}_2$:

$$\mathbb{C}_2 = \{\{1,2\},\{1,3\},\{2,3\},\{1,4,5\},\{1,4,6\},\{1,5,6\},$$
$$\{2,4,5\},\{2,4,6\},\{2,5,6\},\{3,4,5\},\{3,4,6\},\{3,5,6\}\}$$

Note that the 2-coterie $\mathbb{C}_2$ is dominated by the following $\mathbb{C}_1$:

$$\mathbb{C}_1 = \{\{1,2\},\{1,3\},\{2,3\},\{4,5\},\{4,6\},\{5,6\}\}.$$

Furthermore, $\mathbb{C}_1$ can be said to be more resilient to process failure. This can be explained through the example 7, if all processes 1, 2, 3 failed/ corrupted, then a quorum cannot be formed by $\mathbb{C}_2$, but a quorum can still be formed by $\mathbb{C}_1$. For this reason, a nondominated k-coterie provide the availability higher than the other comparative [10].

4. Coterie Join Operation

The coterie join operation is used to combine non empty coteries to construct a new larger coterie.

Definition 6. Let $\mathbb{C}_1$ and $\mathbb{C}_2$ be coteries in the universe U_1 and U_2, respectively, where $U_1 \cap U_2 = \emptyset$. The coterie join operation with input $\mathbb{C}_1$ and $\mathbb{C}_2$ generate a new coterie $\mathbb{C}_3 = T_x(\mathbb{C}_1, \mathbb{C}_2)$ in the universe $U_3 = (U_1 - \{x\}) \cup U_2$, where $\mathbb{C}_3$ is constructed by replacing each process $x \in U_1$ in quorums of $\mathbb{C}_1$ by processes in a quorum of $\mathbb{C}_2$ which is defined as follows [4]:

$$\mathbb{C}_3 = T_x(\mathbb{C}_1,\mathbb{C}_2) = \left\{ Q \middle| Q = \begin{cases} (Q_1 - \{x\}) \cup Q_2 & jika\ x \in Q_1 \\ Q_1 & lainnya \end{cases}, Q_1 \in \mathbb{C}_1, Q_2 \in \mathbb{C}_2 \right\}.$$

Example 8: Let $\mathbb{C}_1$ and $\mathbb{C}_2$ be coteries in the universe $U_1 = \{1,2,3\}$ and $U_2 = \{a,b,c\}$, respectively, as follows:

$$\mathbb{C}_1 = \{\{1,2\},\{1,3\},\{2,3\}\}$$
$$\mathbb{C}_2 = \{\{a,b\},\{a,c\},\{b,c\}\}$$

For $x = 2$, $T_x(\mathbb{C}_1,\mathbb{C}_2) = \mathbb{C}$, where $\mathbb{C}$ is coterie of $U = \{1,3,a,b,c\}$, and $\mathbb{C}$ is constructed by replacing each process x of a quorum in $\mathbb{C}_1$ by quorums in $\mathbb{C}_2$. So, a new coterie $\mathbb{C}$ is obtained

$$\mathbb{C} = T_2(\mathbb{C}_1,\mathbb{C}_2) = \left\{ Q \middle| Q = \begin{cases} (Q_1 - \{2\}) \cup Q_2 & jika\ 2 \in Q_1 \\ Q_1 & lainnya \end{cases}, Q_1 \in \mathbb{C}_1, Q_2 \in \mathbb{C}_2 \right\}.$$

So,

$$\mathbb{C} = \{\{1,a,b\},\{1,a,c\},\{1,b,c\},\{1,3\},\{3,a,b\},\{3,a,c\},\{3,b,c\}\}$$

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5. k-Coterie and Coterie Join Operation

In replacement of a set X on a quorum Q, there are two conditions that may occur is $|X| > |Q|$ or $|X| \leq |Q|$. If $|X| > |Q|$, then the quorum Q is only reduced by several processes in X that is contained in Q. And if $|X| \leq |Q|$, then a quorum can be reduced by X.

Definitions 7. Let $\mathbb{C}_0$ be a k-coterie in the universe U_0 and $\mathbb{C}_1, \dots, \mathbb{C}_m$ be coteries in the universe $U_1, \dots, U_m$, respectively, where $U_i \cap U_j = \emptyset$ for $0 \leq i \neq j \leq m$. The join operation of a k-coterie $\mathbb{C}_0$ and coteries $\mathbb{C}_1, \dots, \mathbb{C}_m$ generate a new k-coterie $\mathbb{C} = T_X(\mathbb{C}_0, \mathbb{C}_1, \dots, \mathbb{C}_m)$ in the universe $U = (U_0 - X) \bigcup_{i=1}^{m} \cup U_i$, where $\mathbb{C}$ is constructed by replacing each process $X = \{x_1, \dots, x_m\} \subseteq U_0$ in quorums of $\mathbb{C}_0$ by processes in a quorum of $\mathbb{C}_1, \dots, \mathbb{C}_m$ which is defined as follows:

$$\mathbb{C} = T_X(\mathbb{C}_0, \mathbb{C}_1, \dots, \mathbb{C}_m)$$

$$= \left\{ W \,\middle|\, W = \begin{cases} (Q_0 - \{x_1, \dots, x_m\}) \bigcup_{j=1}^{m} Q_j\,, & x_1, \dots, x_m \in Q_0 \\ \vdots & \vdots \\ (Q_0 - \{x_1, x_2\}) \cup Q_1 \cup Q_2, & x_1, x_2 \in Q_0; x_3, \dots, x_m \notin Q_0 \\ (Q_0 - \{x_m\}) \cup Q_m, & x_m \in Q_0;\ x_1, \dots, x_{m-1} \notin Q_0 \\ \vdots & \vdots \\ (Q_0 - \{x_2\}) \cup Q_2, & x_2 \in Q_0;\ x_1, x_3, x_4 \dots, x_m \notin Q_0 \\ (Q_0 - \{x_1\}) \cup Q_1, & x_1 \in Q_0;\ x_2, \dots, x_m \notin Q_0 \\ Q_0, & x_1, \dots, x_m \notin Q_0 \end{cases} ,\ Q_0 \in \mathbb{C}_0, Q_1 \in \mathbb{C}_1, \dots, Q_m \in \mathbb{C}_m \right\}$$

Example 9: Let $U_0 = \{1, 2, 3\}$, $U_1 = \{a, b, c\}$, and $U_2 = \{d, e, f\}$ define input coteries $\mathbb{C}_0$ in the universe U_0, $\mathbb{C}_1$ in the universe U_1, and $\mathbb{C}_2$ in the universe U_2 as follows:

$$\mathbb{C}_0 = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$$
$$\mathbb{C}_1 = \{\{a, b\}, \{a, c\}, \{b, c\}\}$$
$$\mathbb{C}_2 = \{\{d, e\}, \{d, f\}, \{e, f\}\}$$

For $X = \{1, 3\}, T_X(\mathbb{C}_0, \mathbb{C}_1, \mathbb{C}_2) = \mathbb{C}$, where $\mathbb{C}$ is a coterie of $U = \{2, a, b, c, d, e, f\}$, and $\mathbb{C}$ is constructed by replacing each process X in quorums of $\mathbb{C}_0$ by processes in a quorum of $\mathbb{C}_1$ and $\mathbb{C}_2$. So, a new coterie $\mathbb{C}$ is obtained

$$\mathbb{C} = T_X(\mathbb{C}_0, \mathbb{C}_1, \mathbb{C}_2)$$

$$= \left\{ W \,\middle|\, W = \begin{cases} (Q_0 - \{1, 3\}) \cup Q_1 \cup Q_2, & 1, 3 \in Q_0 \\ (Q_0 - \{3\}) \cup Q_2, & 3 \in Q_0; 1 \notin Q_0 \\ (Q_0 - \{1\}) \cup Q_1, & 1 \in Q_0; 3 \notin Q_0 \\ Q_0, & 1, 3 \notin Q_0 \end{cases}, Q_0 \in \mathbb{C}_0, Q_1 \in \mathbb{C}_1, Q_2 \in \mathbb{C}_2 \right\}$$

So,

$$\mathbb{C} = \{\{a, b, d, e\}, \{a, b, d, f\}, \{a, b, e, f\}, \{a, c, d, e\}, \{a, c, d, f\}, \{a, c, e, f\}, \{b, c, d, e\}, \{b, c, d, f\}, \{b, c, e, f\}, \{2, a, b\}, \{2, a, c\}, \{2, b, c\}, \{2, d, e\}, \{2, d, f\}, \{2, e, f\}\}$$

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Example 10: Let $U_0 = \{1,2,3,4\}, U_1 = \{a,b,c\}, U_2 = \{d,e,f\}, U_3 = \{g,h,i,j\}$, and $U_4 = \{k,l,m,n\}$ define input 2-coteries $\mathbb{C}_0$ in the universe U_0, $\mathbb{C}_1$ in the universe U_1, $\mathbb{C}_2$ in the universe U_2, $\mathbb{C}_3$ in the universe U_3, and $\mathbb{C}_4$ in the universe U_4 as follows:

$$\mathbb{C}_0 = \{\{1,2\},\{1,3\},\{2,3\},\{4\}\}$$
$$\mathbb{C}_1 = \{\{a,b\},\{a,c\},\{b,c\}\}$$
$$\mathbb{C}_2 = \{\{d,e\},\{d,f\},\{e,f\}\}$$
$$\mathbb{C}_3 = \{\{g,h\},\{g,i\},\{g,j\},\{h,i,j\}\}$$
$$\mathbb{C}_4 = \{\{k,l\},\{k,m\},\{k,n\},\{l,m,n\}\},$$

a. For $X = \{1,2\}, T_X(\mathbb{C}_0,\mathbb{C}_1,\mathbb{C}_2) = \mathbb{C}$, where $\mathbb{C}$ is a 2-koteri of obtained $U = \{3,4,a,b,c,d,e,f\}$, and $\mathbb{C}$ constructed by replacing each process X in quorums of $\mathbb{C}_0$ by processes in a quorum of $\mathbb{C}_1$ and $\mathbb{C}_2$. So, a new 2-koteri $\mathbb{C}$ is obtained

$$\mathbb{C} = T_X(\mathbb{C}_0,\mathbb{C}_1,\mathbb{C}_2)$$
$$= \left\{ W \middle| W = \begin{cases} (Q_0 - \{1,2\}) \cup Q_1 \cup Q_2, & 1,2 \in Q_0 \\ (Q_0 - \{2\}) \cup Q_2, & 2 \in Q_0; 1 \notin Q_0 \\ (Q_0 - \{1\}) \cup Q_1, & 1 \in Q_0; 2 \notin Q_0 \\ Q_0, & 1,2 \notin Q_0 \end{cases}, Q_0 \in \mathbb{C}_0, Q_1 \in \mathbb{C}_1, Q_2 \in \mathbb{C}_2 \right\}$$

So,

$$\mathbb{C} = \{\{a,b,d,e\},\{a,b,d,f\},\{a,b,e,f\},\{a,c,d,e\},\{a,c,d,f\},\{a,c,e,f\},\{b,c,d,e\},\{b,c,d,f\},\{b,c,e,f\},\{3,a,b\},\{3,a,c\},\{3,b,c\},\{3,d,e\},\{3,d,f\},\{3,e,f\},\{4\}\}$$

b. For $X = \{1,2,3,4\}, T_X(\mathbb{C}_0,\mathbb{C}_1,\mathbb{C}_2,\mathbb{C}_3,\mathbb{C}_4) = \mathbb{C}$, where $\mathbb{C}$ is a 2-coterie of $U = \{a,b,c,d,e,f,g,h,i,j,k,l,m,n\}$, and $\mathbb{C}$ constructed by replacing each process X in quorums of $\mathbb{C}_0$ by processes in a quorum of $\mathbb{C}_1,\mathbb{C}_2,\mathbb{C}_3$, and $\mathbb{C}_4$. So, a new 4-coterie $\mathbb{C}$ is obtained

$$\mathbb{C} = T_X(\mathbb{C}_0,\mathbb{C}_1,\mathbb{C}_2,\mathbb{C}_3,\mathbb{C}_4)$$
$$= \left\{ W \middle| W = \begin{cases} (Q_0 - \{2,3\}) \cup Q_2 \cup Q_3, & 2,3 \in Q_0;\ 1,4 \notin Q_0 \\ (Q_0 - \{1,3\}) \cup Q_1 \cup Q_3, & 1,3 \in Q_0;\ 2,4 \notin Q_0 \\ (Q_0 - \{1,2\}) \cup Q_1 \cup Q_2, & 1,2 \in Q_0;\ 3,4 \notin Q_0 \\ (Q_0 - \{4\}) \cup Q_4, & 4 \in Q_0; 1,2,3 \notin Q_0 \end{cases}, Q_0 \in \mathbb{C}_0, Q_1 \in \mathbb{C}_1, Q_2 \in \mathbb{C}_2, Q_3 \in \mathbb{C}_3, Q_4 \in \mathbb{C}_4 \right\}$$

So,

$$\mathbb{C} = \{\{d,e,g,h\},\{d,e,g,i\},\{d,e,g,j\},\{d,e,h,i,j\},\{d,f,g,h\},\{d,f,g,i\},\{d,f,g,j\},\{d,f,h,i,j\},\{e,f,g,h\},\{e,f,g,i\},\{e,f,g,j\},\{e,f,h,i,j\},\{a,b,g,h\},\{a,b,g,i\},\{a,b,g,j\},\{a,b,h,i,j\},\{a,c,g,h\},\{a,c,g,i\},\{a,c,g,j\},\{a,c,h,i,j\},\{b,c,g,h\},\{b,c,g,i\},\{b,c,g,j\},\{b,c,h,i,j\},\{a,b,d,e\},\{a,b,d,f\},\{a,b,e,f\},\{a,c,d,e\},\{a,c,d,f\},\{a,c,e,f\},\{b,c,d,e\},\{b,c,d,f\},\{b,c,e,f\},\{k,l\},\{k,m\},\{k,n\},\{l,m,n\}\}$$

6. The properties of k-Coterie and Coterie Join Operation

Let $\mathbb{C}_0$ is a k-coterie in the universe U_0 and $\mathbb{C}_1,\ldots,\mathbb{C}_m$ is coterie-coterie in the universe $U_1,\ldots,U_m$, respectively, where $U_i \cap U_j = \emptyset$ for $0 \le i \ne j \le m$. Suppose that $\mathbb{C} = T_X(\mathbb{C}_0,\mathbb{C}_1,\ldots,\mathbb{C}_m)$ is in the universe $U = (U_0 - X) \cup U_1 \cup \ldots \cup U_m$, where $X = \{x_1,\ldots,x_m\} \subseteq U_0$.

Theorem 3. $\mathbb{C}$ is a k-coterie in the universe U.

Proof:

We will show that $\mathbb{C}$ satisfies the three conditions of a k-coterie in the definition 4.

1) Non-intersection property

$\mathbb{C}_0$ is a k-coterie means $\mathbb{C}_0$ satisfies a non-intersection property, that is for any $h\,(1 \le h < k)$ quorum $Q_{0,1}, \dots, Q_{0,h} \in \mathbb{C}_0$, $Q_{0,i} \cap Q_{0,j} = \emptyset$ for all i, j, $1 \le i \ne j \le h$, there is a quorum $Q_0 \in \mathbb{C}_0$ so that $Q_0 \cap Q_{0,i} = \emptyset$ for all $i, 1 \le i \le h$.

Taken any h disjoint quorums $W_1, \dots, W_h \in \mathbb{C}$ that is constructed by replacing each process $X = \{x_1, \dots, x_a, \dots, x_b, \dots, x_m\} \subseteq U_0$ within each h disjoint quorum $Q_{0,1}, \dots, Q_{0,h} \in \mathbb{C}_0$ by quorums $Q_1 \in \mathbb{C}_1, Q_2 \in \mathbb{C}_2, \dots Q_a \in \mathbb{C}_a, \dots, Q_b \in \mathbb{C}_b, \dots, Q_m \in \mathbb{C}_m$.

If then$|X| \le |Q_{0,i}|$

$$\begin{aligned} W_1 &= (Q_{0,1} - X) \bigcup_{i=1}^{m} Q_i \\ W_2 &= Q_{0,2} \\ &\vdots \\ W_h &= Q_{0,h} \end{aligned}$$

And if, then$|X| > |Q_{0,i}|$

$$\begin{aligned} W_1 &= (Q_{0,1} - \{x_1, \dots, x_a\}) \bigcup_{i=1}^{a} Q_i \\ &\vdots \\ W_m &= (Q_{0,m} - \{x_b, \dots, x_m\}) \bigcup_{i=b}^{m} Q_i \\ W_{m+1} &= Q_{0,m+1} \\ &\vdots \\ W_h &= Q_{0,h} \end{aligned}$$

Because of $U_i \cap U_j = \emptyset$ for all i, j, $0 \le i \ne j \le m$, then $Q_i \cap Q_j = \emptyset$ for all i, j, $0 \le i \ne j \le m$. Therefore $Q_{0,i} \cap Q_{0,j} = \emptyset$ for all $i, j, 1 \le i \ne j \le h$, then $W_i \cap W_j = \emptyset$ for all $i, j, 1 \le i \ne j \le h$. There is a quorum $W = Q_0 \in \mathbb{C}$ so $W \cap W_i = Q_0 \cap Q_{0,i} = \emptyset$ for all $i, j, 1 \le i \ne j \le h$. So, for each quorum $W_1, \dots, W_h \in \mathbb{C}$ has non-intersection property.

2) Intersection property

$\mathbb{C}_0$ is a k-coterie means $\mathbb{C}_0$ satisfies the intersection property because there is only k disjoint quorums in $\mathbb{C}_0$.That is, for any $l > k$ quorums $Q_{0,1}, \dots, Q_{0,l} \in \mathbb{C}_0$, there is a pair of quorums $Q_{0,i}, Q_{0,j}$ such that $Q_{0,i} \cap Q_{0,j} \ne \emptyset$.

Taken any l quorum $W_1, \dots, W_l \in \mathbb{C}$ that is constructed by replacing each process of $X = \{x_1, \dots, x_m\} \subseteq U_0$ in any l quorum $Q_{0,1}, \dots, Q_{0,l} \in \mathbb{C}_0$ by quorums $Q_1 \in \mathbb{C}_1, Q_2 \in \mathbb{C}_2, \dots, Q_m \in \mathbb{C}_m$.

Therefore, there is a pair of quorums $Q_{0,i}, Q_{0,j}$ such that $Q_{0,i} \cap Q_{0,j} \ne \emptyset$, then there is a pair of quorums W_i, W_j such that $W_i \cap W_j \ne \emptyset$. Thus, for any l quorum $W_1, \dots, W_l \in \mathbb{C}$ has intersection property.

3) minimality

$\mathbb{C}_i$ is k-coterie means $\mathbb{C}_i$ satisfies minimality, that is for any $Q_{i,1}, Q_{i,2} \in \mathbb{C}$, $Q_{i,1} \nsubseteq Q_{i,2}$ for $0 \le i \le m$. Suppose that $Q_0 \in \mathbb{C}_0, Q_1 \in \mathbb{C}_1, \dots, Q_m \in \mathbb{C}_m$. Because of $U_i \cap U_j = \emptyset$ for all i, j, $0 \le i \ne j \le m$, then $Q_i \cap Q_j = \emptyset$ for all i, j, $0 \le i \ne j \le m$. So, $Q_i \nsubseteq Q_j$ for $0 \le i \ne j \le m$. For each $Q_{0,i}, Q_{0,j} \in \mathbb{C}_0$ is constructed any quorum $W_i, W_j \in \mathbb{C}$, which is as follows

$$W_i = (Q_{0,i} - X) \bigcup_{a=1}^{m} Q_a,$$

$$W_j = (Q_{0,j} - X) \cup \bigcup_{b=1}^{m} Q_b,$$

where $X = \{x_1, \dots, x_m\}$ and $Q_1 \in \mathbb{C}_1, Q_2 \in \mathbb{C}_2, \dots, Q_m \in \mathbb{C}_m$.

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Therefore $Q_{0,1} \nsubseteq Q_{0,2} Q_{1,1} \nsubseteq Q_{1,2}, \ldots, Q_{m,1} \nsubseteq Q_{m,2} Q_i \nsubseteq Q_j$ for $0 \le i \ne j \le m$, then $W_i \nsubseteq W_j$. Thus, no quorum is a subset or equal to another quorum.

Thus, three conditions of a k-coterie are satisfied, then $\mathbb{C}$ is a k-coterie in the universe U.
Theorem 4. If $\mathbb{C}_0, \mathbb{C}_1, \ldots, \mathbb{C}_m$ is nondominated, $\mathbb{C}$ is nondominated.
Proof:
Assume that $\mathbb{C}$ is dominated that means there is a set $R \subseteq U$ such that

1. $\forall W \in \mathbb{C}, W \nsubseteq R$ and
2. for any k collection of disjoint quorums $\{W_1, \ldots, W_k\} \subseteq \mathbb{C}$, $W_i \cap R \ne \emptyset$ for $1 \le i \le k$.

Because $\mathbb{C}_0$ is nondominated that mean there is a quorum $Q_0 \in \mathbb{C}_0$ such that $Q_0 \subseteq R_0$. Likewise, $\mathbb{C}_i$ is nondominated that mean there is a quorum $Q_i \in \mathbb{C}_i$ such that $Q_i \subseteq R_i$, where $R_i = R \cap U_i$ for all $i, 1 \le i \le m$. An analogy R_0 is adjusted by the existence of a quorum of coterie $\mathbb{C}_1$, ..., or $\mathbb{C}_m$ inside R.

To obtain a contradiction, we will show that there is a quorum $W' \in \mathbb{C}$ such that $W' \subseteq R$.

1. For $x_1, \ldots, x_m \notin Q_0$. Suppose $R_0 = R \cap U_0$. Suppose $W' = Q_0$, then $W' \in \mathbb{C}$. Because $Q_0 \subseteq R_0$ and $R_0 \subseteq R$, this shows that $W' \subseteq R$.
2. For $A \subseteq X$, $A \subseteq Q_0$, and $(X - A) \cap Q_0 = \emptyset$. Suppose $R_0 = (R \cup A) \cap U_0$. Suppose $W' = (Q_0 - A) \cup_i Q_i$, then $W' \in \mathbb{C}$. Since $Q_0 \subseteq R_0, Q_i \subseteq R_i$, and $(R_0 - X) \cup_i R_i \subseteq R$, this shows that $W' \subseteq R$.

Therefore, there is a quorum $W' \in \mathbb{C}$ such that $W' \subseteq R$. Thus, $\mathbb{C}$ is nondominated.

7. The Analysis of Availability

The availability of a coterie is a chance that at least one procesa is still able to access the resource. While the availability of k-coterie is an opportunity that at most k processes can access resources, where $k \ge 1$.

Definitions 8. Suppose $\mathbb{C}$ is a k-coterie in the universe U, and α $(1 \le \alpha \le k)$ are integers. The (k, α)-characteristic function $F_{\mathbb{C},k,\alpha}$ of $\mathbb{C}$ is a function from 2^U to $\{0,1\}$ defined as follows:
For each $S \subseteq U$, $F_{\mathbb{C},k,\alpha} = 1$ if and only if there is α quorum $Q_1, \ldots, Q_\alpha \in \mathbb{C}$ satisfy both of the following conditions:

1) $Q_i \cap Q_j = \emptyset$ *for and* $1 \le i, j \le \alpha,\ i \ne j$,
2) $\forall i,\ \ Q_i \subseteq S$.

Definitions 9. Suppose $\mathbb{C}$ is a k-coterie and α $(1 \le \alpha \le k)$ be an integer. The (k, α)-availability $A_{k,\alpha}(\mathbb{C})$ is the probability that at least α processes can enter a critical section. Let S be the set of processes that be operating. The probability that the set of processes that be operating exactly S is $p^{|S|}(1-p)^{n-|S|}$. So, the (k, α)-availability of a coterie $\mathbb{C}$ is [7]

$$A_{k,\alpha}(\mathbb{C}) = \sum_{S \subseteq U} F_{\mathbb{C},k,\alpha}(S) p^{|S|}(1-p)^{n-|S|}.$$

Contrast $A_{k,\alpha}(\mathbb{C})$ is defined dual availability coterie $\mathbb{C}$, $DA_{k,\alpha}(\mathbb{C})$, as the probability that $\bar{S}$ consists of α quorums in $\mathbb{C}$. Because of that is obtained [11]

$$DA_{k,\alpha}(\mathbb{C}) = \sum_{S \subseteq U} F_{\mathbb{C},k,\alpha}(\bar{S}) p^{|S|}(1-p)^{n-|S|}.$$

Based on availability, a dominated k-coterie is revised as follows.
Theorem 5. A k-coterie $\mathbb{C}$ in the universe U is dominated if and only if $\exists H \subseteq U$ such that the following properties are satisfied:

1) $\forall Q \in \mathbb{C}, Q \nsubseteq H$,
2) For any collection of k disjoint quorums $\{Q_1, Q_2, \ldots, Q_k\} \subseteq \mathbb{C}, Q_j \nsubseteq \bar{H}$ for $1 \le j \le k$

Through the availability requirement, it can be shown that k-coterie is nondominated.

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Theorem 6. A k-coterie $\mathbb{C}$ is nondominated if and only if $\forall p, A_{k,\alpha=1}(\mathbb{C}) + DA_{k,\alpha=k}(\mathbb{C}) = 1$.
Proof:
We will show that if a k-coterie $\mathbb{C}$ is nondominated then $\forall p, A_{k,\alpha=1}(\mathbb{C}) + DA_{k,\alpha=k}(\mathbb{C}) = 1$. By contraposition, supposing $\exists p, A_{k,\alpha=1}(\mathbb{C}) + DA_{k,\alpha=k}(\mathbb{C}) \neq 1$.
It is known that $\forall S \subseteq U$, one of $F(S) = 1$ or $F(\bar{S}) = 1$ but not both.
Therefore,

$$A_{k,\alpha=1}(\mathbb{C}) + DA_{k,\alpha=k}(\mathbb{C}) = \sum_{S\subseteq U}\left[F_{\mathbb{C},k,\alpha=1}(S) + F_{\mathbb{C},k,\alpha=k}(\bar{S})\right]p^{|S|}(1-p)^{n-|S|} \leq \sum_{S\subseteq U} p^{|S|}(1-p)^{n-|S|}$$

Because

$$\forall p, \sum_{S\subseteq U} p^{|S|}(1-p)^{n-|S|} = 1,$$

If $A_{k,\alpha=1}(\mathbb{C}) + DA_{k,\alpha=k}(\mathbb{C}) \neq 1$ then $\exists S \subseteq U, F(S) = 0 \wedge F(\bar{S}) = 0$.
That is,
$\exists S \subseteq U, (\forall Q \in \mathbb{C}, Q \nsubseteq S) \wedge$ (For any collection of disjoint k quorums $\{Q_1, Q_2, \dots, Q_k\} \subseteq \mathbb{C}, Q_j \nsubseteq \bar{S}$ for $1 \leq j \leq k$)
From theorem 5 is known that $\mathbb{C}$ is dominated.
Furthermore, we will show that if $\forall p, A_{k,\alpha=1}(\mathbb{C}) + DA_{k,\alpha=k}(\mathbb{C}) = 1$, then $\mathbb{C}$ is nondominated. By contraposition, suppose $\mathbb{C}$ is dominated. Then, by Theorem 5, there is a set H that satisfy $(\forall Q \in \mathbb{C}, Q \nsubseteq H) \wedge$ (For any collection of disjoint k quorums $\{Q_1, Q_2, \dots, Q_k\} \subseteq \mathbb{C}, Q_j \nsubseteq \bar{H}$ for $1 \leq j \leq k$) that is $F(H) = F(\bar{H}) = 0$. As the result, $\forall p$,

$$A_{k,\alpha=1}(\mathbb{C}) + DA_{k,\alpha=k}(\mathbb{C}) = \sum_{S\subseteq U}\left[F_{\mathbb{C},k,\alpha=1}(S) + F_{\mathbb{C},k,\alpha=k}(\bar{S})\right]p^{|S|}(1-p)^{n-|S|}$$
$$\neq \sum_{S\subseteq U} p^{|S|}(1-p)^{n-|S|} = 1.$$

Example 11: From example 10a is obtained a new 2-coterie
$\mathbb{C} = \{\{a,b,d,e\},\{a,b,d,f\},\{a,b,e,f\},\{a,c,d,e\},\{a,c,d,f\},\{a,c,e,f\},\{b,c,d,e\},\{b,c,d,f\},$
$\{b,c,e,f\},\{3,a,b\},\{3,a,c\},\{3,b,c\},\{3,d,e\},\{3,d,f\},\{3,e,f\},\{4\}\}$
For $p = 0.1$,

$$\begin{aligned} A_{2,1}(\mathbb{C}) &= \sum_{S\subseteq U} F_{\mathbb{C},2,1}(S)p^{|S|}(1-p)^{n-|S|} \\ &= (0.1)^1(0.9)^7 + 7(0.1)^2(0.9)^6 + 27(0.1)^3(0.9)^5 + 64(0.1)^4(0.9)^4 \\ &\quad +56(0.1)^5(0.9)^3 + 28(0.1)^6(0.9)^2 \\ &\quad +8(0.1)^7(0.9)^1 + (0.1)^8 \\ &= 0.04782969 + 0.03720087 + 0.01594323 + 0.00419904 + 0.00040824 \\ &\quad +0.00002268 + 0.00000072 + 0.00000001 \\ &= 0.10560448 \end{aligned}$$

$$\begin{aligned} DA_{2,2}(\mathbb{C}) &= \sum_{S\subseteq U} F_{\mathbb{C},2,2}(\bar{S})p^{|S|}(1-p)^{n-|S|} \\ &= (0.9)^8 + 7(0.1)^1(0.9)^7 + 21(0.1)^2(0.9)^6 + 29(0.1)^3(0.9)^5 + 6(0.1)^4(0.9)^4 \\ &= 0.43046721 + 0.33480783 + 0.11160261 + 0.01712421 + 0.00039366 \\ &= 0.89439552 \end{aligned}$$

Furthermore, Table 1. shows $A_{2,1}(\mathbb{C}) + DA_{2,2}(\mathbb{C}) = 1$ from example 11.

Table 1. The Sum of $(2,1)$ -Availability and (2, 2)-Dual Availability of $\mathbb{C}$

p	$1-p$	$A_{2,1}(\mathbb{C})$	$DA_{2,2}(\mathbb{C})$	$A_{2,1}(\mathbb{C})+DA_{2,2}(\mathbb{C})$
0	1	0.00000000	1.00000000	1.00000000
0.1	0.9	0.10560448	0.89439552	1.00000000
0.2	0.8	0.23847168	0.76152832	1.00000000
0.3	0.7	0.40378368	0.59621632	1.00000000
0.4	0.6	0.58382848	0.41617152	1.00000000
0.5	0.5	0.75000000	0.25000000	1.00000000
0.6	0.4	0.87744768	0.12255232	1.00000000
0.7	0.3	0.95552128	0.04447872	1.00000000
0.8	0.2	0.99038208	0.00961792	1.00000000
0.9	0.1	0.99937728	0.00062272	1.00000000
1	0	1.00000000	0.00000000	1.00000000

Therefore $\forall p, A_{2,1}(\mathbb{C}) + DA_{2,2}(\mathbb{C}) = 1$, $\mathbb{C}$ is nondominated. This is true vice versa.

8. Conclusion

The k-coterie and coterie join operation is provided to build a new larger k-coterie, $\mathbb{C}$, so that the availability of k-coterie increases. Construction of a new k-coterie is done by replacing the set of m processes in the quorum in k-coterie by processes of quorums in coterie from the universes that do not intersect. Through analysis of availability, it is obtained that k-coterie $\mathbb{C}$ is nondominated if and only if $\forall p, A_{k,\alpha=1}(\mathbb{C}) + DA_{k,\alpha=k}(\mathbb{C}) = 1$.

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